Application of the Proper Generalized Decomposition to Solve MagnetoElectric Problem

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Among the model order reduction techniques, the Proper Generalized Decomposition (PGD) has shown its efficiency to solve a large number of engineering problems. In this communication, the PGD approach is applied to solve a multi-physics problem based on a magnetoelectric device.

Index Terms— Finite element method, Magnetoelectric problem, Proper Generalized Decomposition.

I. INTRODUCTION

To REDUCE the computation time of numerical models in the time or frequency domain, Model Order Reduction (MOR) methods have been developed and presented in the literature. These approaches have been mainly used to study a large number of devices in mechanics. In this field, the Proper Generalized Decomposition method has been largely studied [1][2]. In computational electromagnetics, the PGD approach has been extended to solve static or quasi-static problems [3][7]. The principle of the PGD method consists in expressing the solution by a sum of functions depending on each parameter of the problem, so-called modes. Each mode is determined by an iterative procedure and depends on the previous modes. In the case of systems of partial differential equations in the frequency domain, the PGD approach approximates the solution by a sum of functions separable in frequency and space.

In this communication, we propose to apply the PGD approach with a magnetoelectric problem. The results obtained with the PGD model are compared in terms of accuracy and computation time with a classical approach.

II. MAGNETOELECTRIC PROBLEM

Let us consider a magnetoelectric problem based on a 2D sensor composed of magnetostrictive (MM) and piezoelectric (PZT) materials. An external harmonic magnetic field $\mathbf{H}_{ext}$ is imposed. By neglecting the charge density and the external forces, the system of equations to solve is

$$\text{div}\mathbf{T} + \rho\omega^2\mathbf{u} = 0 \tag{1}$$
$$\text{div}\mathbf{D} = 0 \tag{2}$$
$$\mathbf{curl}\,\mathbf{H} = 0 \tag{3}$$

with $\mathbf{T}$ the stress tensor, $\mathbf{u}$ the displacement, $\mathbf{D}$ the electric induction, $\mathbf{H}$ the magnetic field, ρ the mass density and ω the angular frequency. The constitutive laws of the MM and PZT materials are

$$\mathbf{T} = c\mathbf{S} - \tau^t\mathbf{E} - e^t\mathbf{B} \tag{4}$$
$$\mathbf{D} = \varepsilon\mathbf{E} + \tau\mathbf{S} \tag{5}$$
$$\mathbf{H} = \nu\mathbf{B} - e\mathbf{S} \tag{6}$$

with $\mathbf{S}$ the strain tensor, $\mathbf{E}$ the electric field, c the stiffness tensor, e the magnetostrictive coefficients, τ the piezoelectric coefficients, ε the electric permittivity and ν the magnetic reluctivity. To solve the problem, a formulation in term of potentials can be used. $\mathbf{E}$ and $\mathbf{B}$ are expressed such that $\mathbf{E}=-\mathbf{grad}\,v$ and $\mathbf{B}=\mathbf{curl}\,\mathbf{A}$ with v the electric potential and $\mathbf{A}$ the magnetic potential. The strain tensor $\mathbf{S}$ is given by $\mathbf{S}=1/2(\mathbf{grad}\,\mathbf{u} + \mathbf{grad}^t\,\mathbf{u})$. Then, we seek for the solutions v, $\mathbf{A}$ and $\mathbf{u}$ in the space domain D and in the angular frequency interval $[\omega_{min}:\omega_{max}]$. The quantities of interest are the voltage U between the two electrodes of the PZT layer and the maximal deformations of the sensor.
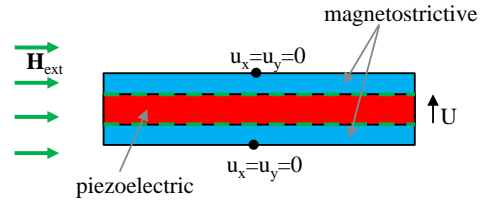

Fig. 1. Magnetoelectric sensor

III. PGD FORMULATION

The PGD method consists in approximating the solutions by sums of separable functions in frequency and space. Then, v, $\mathbf{A}$ and $\mathbf{u}$ are approximated by separated forms of space and frequency functions,

$$v \approx \sum_{j=1}^{M} R_j^v(\mathbf{x})S_j^v(\omega),\ \mathbf{A} \approx \sum_{j=1}^{M} \mathbf{R}_j^A(\mathbf{x})S_j^A(\omega)\ \text{and}\ \mathbf{u} \approx \sum_{j=1}^{M} \mathbf{R}_j^u(\mathbf{x})S_j^u(\omega)$$

with $\mathbf{x}\in D$, $\omega\in[\omega_{min}:\omega_{max}]$ and M the number of modes of the expansions. To apply the PGD approach, we consider a weak formulation on $D\times[\omega_{min}:\omega_{max}]$ of (1), (2) and (3). Then, we have:

$$\int_{\omega_{min}}^{\omega_{max}}\int_D \mathbf{u'}\cdot[\text{div}\mathbf{T} + \rho\omega^2\mathbf{u}]dDd\omega = 0 \tag{7}$$

$$\int_{\omega_{min}}^{\omega_{max}}\int_D v'\cdot\text{div}\mathbf{D}\,dDd\omega = 0 \tag{8}$$

$$\int_{\omega_{min}}^{\omega_{max}}\int_D \mathbf{A'}\cdot\mathbf{curl}\mathbf{H}\,dDd\omega = 0 \tag{9}$$

To compute the set of functions R_j^l and S_j^l for $j\in[1:M]$ and $l=\{v, A, u\}$, an iterative enrichment approach is used. At the nth

iteration, the functions R_n^1 and S_n^1 are expressed as a function of the previous functions R_i^1 and S_i^1 with $i \in [1:n-1]$. Two sets of equations deduced from (7), (8) and (9) are solved iteratively. In a first step, we assume that the functions S_n^1 with $l = \{v, A, u\}$ are known in order to calculate the functions R_n^1. In a second step, the functions S_n^1 are recomputed with the known functions R_n^1. The two steps are repeated until convergence of all functions R_n^1 and S_n^1.

IV. APPLICATION

In term of application, we consider the device presented in Fig. 1. The 2D mesh is composed of 3283 nodes and 6525 triangles. The frequency interval of simulation is fixed at $[10^4;10^5]$Hz with 401 equidistributed discrete values. The quantities of interest are the voltage between the two electrodes and the maximal deformation according to the axis y. Fig. 2 and 3 present the evolutions of the voltage magnitude and of the maximal deformation versus the frequency obtained from a "classic" finite element model (reference) and from the PGD model for different number of modes. With a low number of modes, the waveform of the voltage magnitude versus the frequency is close to the reference, with M=2, the relative error is close to 0.1%. To obtain a good approximation of the maximal deformation, the number of modes must be greater. In our case, with M=13, we obtain a good approximation. In Fig. 3, we can observe that the maximal deformations on the high frequencies are captured with a low number of modes. The number of modes must be increased to capture the low frequency maximal deformations. If the quantity of interest is the voltage between the two electrodes, the PGD gives a good approximation with M = 2. In this case, the speed up is 34 versus the classical approach. To obtain mechanical deformations close to the references, the modes number of the PGD solutions must be greater than 15, the speed up is then 4.3. Fig. 4 gives the deformation of the structure at the resonance frequency obtained from the PGD with M=15. Fig. 5 presents the difference of the deformation at the resonance frequency obtained with the classical approach and the PGD approximation. The values of the difference are much lower compared to the mechanical deformation in Fig. 4. In term of perspective, the PGD will be applied with a magnetoelectric actuator coupled with an electric equation to take into account the supply.
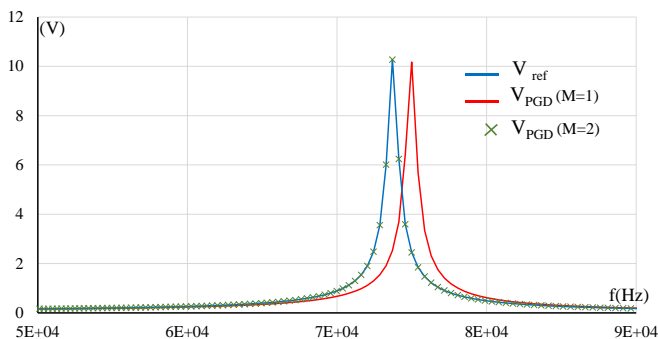

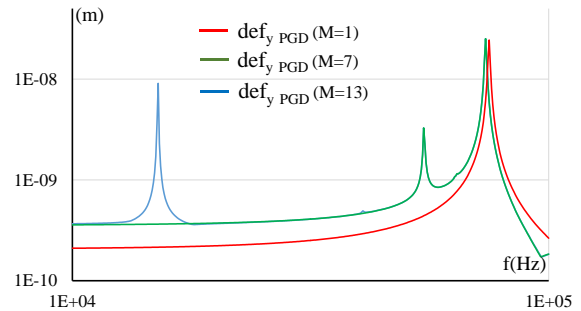Fig. 2. Magnitude of the voltage versus the frequency.

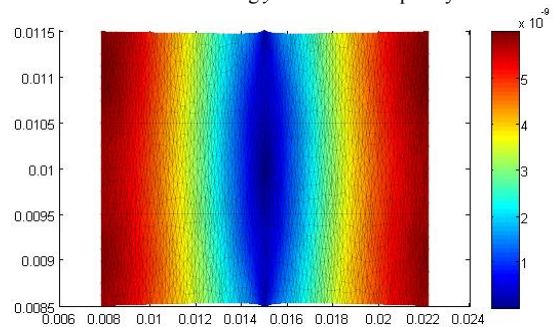Fig. 3. Maximal deformation along y versus the frequency.

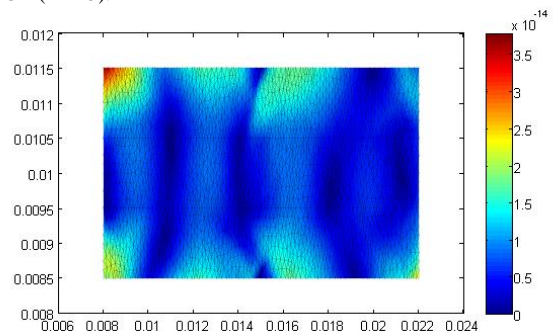Fig. 4. Deformation of the structure at the resonance frequency obtained from the PGD (M=15).

Fig. 5. Difference of the deformation at the resonance frequency obtained from the reference and the PGD approximation.

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